

問題 次の式を因数分解せよ。

(1) a^4+4

(2) a^4+a^2+1

(11) $(a-b)^3+(b-c)^3+(c-a)^3$

(3) $(ab+a+b)(a+b+1)-ab$

(4) $(ab+1)(a+1)(b+1)+ab$

(12) $(a+b+c)^3-a^3-b^3-c^3$

(5) $bc(b-c)+ca(c-a)+ab(a-b)$

(6) $(a+b)(b+c)(c+a)+abc$

(13) $2(b^2c^2+c^2a^2+a^2b^2)-(a^4+b^4+c^4)$

(7) $a^2(b+c)+b^2(c+a)+c^2(a+b)+3abc$

(8) $(a+b+c)(bc+ca+ab)-abc$

(9) $a(b+c)^2+b(c+a)^2+c(a+b)^2-4abc$

(14) $a^3(b-c)+b^3(c-a)+c^3(a-b)$

(10) $a^3+b^3+c^3-3abc=a^3+(b+c)^3-3(b+c)bc-3abc$

挑戦してみたい因数分解の問題

問題 次の式を因数分解せよ。

$$\begin{aligned}
 (1) \quad a^4 + 4 &= a^4 + 4a^2 + 4 - 4a^2 \\
 &= (a^2 + 2)^2 - (2a)^2 \quad \leftarrow A^2 - B^2 \text{の形} \\
 &= (a^2 + 2a + 2)(a^2 - 2a + 2)
 \end{aligned}$$

$$\begin{aligned}
 (2) \quad a^4 + a^2 + 1 &= a^4 + 2a^2 + 1 - a^2 \\
 &= (a^2 + 1)^2 - a^2 \quad \leftarrow A^2 - B^2 \text{の形} \\
 &= (a^2 + a + 1)(a^2 - a + 1)
 \end{aligned}$$

$$\begin{aligned}
 (3) \quad (ab + a + b)(a + b + 1) - ab & \quad a \text{ について整理} \\
 &= \{(b+1)a + b\} \{a + (b+1)\} - ab \quad \leftarrow \downarrow \\
 &= (b+1)a^2 + \{(b+1)^2 + b - b\}a + b(b+1) \\
 &= (b+1)\{a^2 + (b+1)a + b\} \\
 &= (a+b)(a+1)(b+1)
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad (ab+1)(a+1)(b+1) + ab & \\
 &= (ab+1)\{(ab+1) + (a+b)\} + ab \\
 &= (ab+1)^2 + (a+b)(ab+1) + ab \quad \leftarrow \downarrow \\
 &= \{(ab+1) + a\} \{(ab+1) + b\} \quad (ab+1) \text{ について整理} \\
 &= (ab+a+1)(ab+b+1)
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad bc(b-c) + ca(c-a) + ab(a-b) & \\
 &= a^2b - a^2c - ab^2 + ac^2 + bc(b-c) \\
 &= (b-c)a^2 - (b^2 - c^2)a + bc(b-c) \\
 &= (b-c)\{a^2 - (b+c)a + bc\} \\
 &= -(b-c)(c-a)(a-b)
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad (a+b)(b+c)(c+a) + abc & \\
 &= (b+c)\{a^2 + (b+c)a + bc\} + abc \\
 &= (b+c)a^2 + \{(b+c)^2 + bc\}a + bc(b+c) \\
 &= \{a + (b+c)\} \{(b+c)a + bc\} \\
 &= (a+b+c)(bc+ca+ab)
 \end{aligned}$$

$$\begin{aligned}
 (7) \quad a^2(b+c) + b^2(c+a) + c^2(a+b) + 3abc & \\
 &= (b+c)a^2 + (b^2 + 3bc + c^2)a + bc^2 + b^2c \\
 &= (b+c)a^2 + \{(b+c)^2 + bc\}a + bc(b+c) \\
 &= \{a + (b+c)\} \{(b+c)a + bc\} \\
 &= (a+b+c)(bc+ca+ab)
 \end{aligned}$$

$$\begin{aligned}
 (8) \quad (a+b+c)(bc+ca+ab) - abc & \\
 &= \{a + (b+c)\} \{(b+c)a + bc\} - abc \\
 &= (b+c)a^2 + \{(b+c)^2 + bc - bc\}a + bc(b+c) \\
 &= (b+c)\{a^2 + (b+c)a + bc\} \\
 &= (b+c)(c+a)(a+b)
 \end{aligned}$$

$$\begin{aligned}
 (9) \quad a(b+c)^2 + b(c+a)^2 + c(a+b)^2 - 4abc & \\
 &= (b+c)^2a + b(a^2 + 2ac + c^2) + c(a^2 + 2ab + b^2) - 4abc \\
 &= (b+c)a^2 + (b+c)^2a + bc(b+c) \\
 &= (b+c)(c+a)(a+b)
 \end{aligned}$$

$$\begin{aligned}
 (10) \quad a^3 + b^3 + c^3 - 3abc &= a^3 + (b+c)^3 - 3(b+c)bc - 3abc \\
 &= \{a + (b+c)\} \{a^2 - a(b+c) + (b+c)^2\} - 3(a+b+c)bc \\
 &= (a+b+c)\{a^2 - a(b+c) + (b+c)^2 - 3bc\} \\
 &= (a+b+c)(a^2 + b^2 + c^2 - bc - ca - ab)
 \end{aligned}$$

$$\begin{aligned}
 (11) \quad (a-b)^3 + (b-c)^3 + (c-a)^3 & \quad \sqrt{A^3 + B^3} = (A+B)(A^2 - AB + B^2) \\
 &= \{(a-b) + (b-c)\} \{(a-b)^2 - (a-b)(b-c) + (b-c)^2\} + (c-a)^3 \\
 &= -(c-a)\{(a-b)^2 - (a-b)(b-c) + (b-c)^2 - (c-a)^2\} \quad \leftarrow \begin{matrix} (b-c)^2 - (c-a)^2 \\ = (b-a)(a+b-2c) \end{matrix} \\
 &= -(c-a)\{(a-b)^2 - (a-b)(b-c) + (b-a)(a+b-2c)\} \\
 &= -(c-a)(a-b)\{(a-b) - (b-c) - (a+b-2c)\} \\
 &= -(c-a)(a-b)(3c-3b) \\
 &= 3(b-c)(c-a)(a-b)
 \end{aligned}$$

$$\begin{aligned}
 (12) \quad (a+b+c)^3 - a^3 - b^3 - c^3 & \quad \sqrt{A^3 - B^3} = (A-B)(A^2 + AB + B^2) \\
 &= \{(a+b+c) - a\} \{(a+b+c)^2 + (a+b+c)a + a^2\} - (b^3 + c^3) \\
 &= (b+c)\{3a^2 + 3(b+c)a + (b+c)^2\} - (b+c)(b^2 - bc + c^2) \\
 &= (b+c)\{3a^2 + 3(b+c)a + (b+c)^2 - (b^2 - bc + c^2)\} \quad \leftarrow \begin{matrix} (b+c)^2 - (b^2 - bc + c^2) \\ = 3bc \end{matrix} \\
 &= 3(b+c)\{a^2 + (b+c)a + bc\} \\
 &= 3(b+c)(c+a)(a+b)
 \end{aligned}$$

$$\begin{aligned}
 (13) \quad 2(b^2c^2 + c^2a^2 + a^2b^2) - (a^4 + b^4 + c^4) & \\
 &= -a^4 + 2(b^2 + c^2)a^2 - (b^4 - 2b^2c^2 + c^4) \\
 &= -a^4 + (2b^2 + 2c^2)a^2 - (b^2 - c^2)^2 \\
 &= -a^4 + (2b^2 + 2c^2)a^2 - (b+c)^2(b-c)^2 \\
 &= -a^4 + \{(b+c)^2 + (b-c)^2\}a^2 - (b+c)^2(b-c)^2 \\
 &= -\{a^2 - (b+c)^2\} \{a^2 - (b-c)^2\} \\
 &= -\{a + (b+c)\} \{a - (b+c)\} \{a + (b-c)\} \{a - (b-c)\} \\
 &= (a+b+c)(-a+b+c)(a-b+c)(a+b-c)
 \end{aligned}$$

$$\begin{aligned}
 (14) \quad a^3(b-c) + b^3(c-a) + c^3(a-b) & \\
 &= (b-c)a^3 - (b^3 - c^3)a + (b^2 - c^2)bc \quad \leftarrow a \text{ について整理} \\
 &= (b-c)\{a^3 - (b^2 + bc + c^2)a + (b+c)bc\} \\
 &= (b-c)\{(c-a)b^2 + (c^2 - ca)b + (a^3 - c^2a)\} \quad \leftarrow \{ \} \text{内を } b \text{ について整理} \\
 &= (b-c)\{(c-a)b^2 + (c-a)bc - (c^2 - a^2)a\} \\
 &= (b-c)(c-a)\{b^2 + bc - (c+a)a\} \\
 &= (b-c)(c-a)\{(b-a)c + (b^2 - a^2)\} \quad \leftarrow \{ \} \text{内を } c \text{ について整理} \\
 &= -(b-c)(c-a)(a-b)(a+b+c)
 \end{aligned}$$